VIII. *Astronomical Observations. In two Letters from M.* Francis de Zach, *Professor of Mathematics, and Member of the Royal Academies of Sciences at* Marseilles, Dijon, *and* Lyons, *to Mr.* Tiberius Cavallo, *F. R. S.*

Read December 23, 1784.

SIR, Lyons, April 4, 1783.

I SEND you the account of the observations on the eclipse of the moon, which I have made together with the rev. Father LE FEVRE, Astronomer at Lyons, in the Observatory called *au grand Collège*; to which I shall add the observations of the vernal equinox; some observations on Jupiter's satellites, made at Marseilles by M. SAINT JAQUES DE SYLVABELLE; and, lastly, a new solution of a problem that occurs in computing the orbits of comets. If you think that these observations do in any way deserve the notice of the Royal Society, I shall be very glad you would communicate them. In order to ascertain the going of the pendulum clock, I took several corresponding altitudes of the sun, which you will find in the following table. On the day of the eclipse the sky was very serene, nothing could be finer, and it continued so during the observation. I determined to use an achromatic telescope of $3\frac{1}{2}$ feet length, that shews objects in their natural position, because the diluted and uncertain termination of the true shadow of the earth appears more perfectly defined by small than by large telescopes, which magnify too much, and give too great a transit between the penumbra and the true dark shadow. On that account some celebrated astronomers advise to use for the eclipses of the moon no greater telescopes than of four or five feet length. It was remarked at Paris, that in an eclipse of the moon, observed through a telescope of DOLLOND, the focus of its object lens being 30 inches, and likewise through a telescope of five feet length; the eclipse appeared to begin 4′ 7″ sooner, and to end 4′ 7″ later, through the small than through the long telescope; the like has been remarked by several others, and it has been also observed by myself. As to my observations I am tolerably satisfied with them, as they do not differ materially from those of Father LE FEVRE, though it is known that in eclipses of the moon no greater exactness than that of a minute can be obtained. The moon's spots were carefully observed; for it is known, that the mean of the observations of the moon's spots is sufficient to ascertain the longitude of a place to 4″ or 5″ nearly. M. DE LA LANDE comparing the observations of the moon's spots in an eclipse, made the 22d of November, 1760, in Vienna, by the Imperial Astronomer Abbé HELL, with those made at the same time in Paris by M. MESSIER, finds the difference of meridians to be 56′ 13″, which agrees very exactly with that ascertained by other means.

Correfpondent altitudes of the Sun taken with a quadrant of three-feet radius, in order to adjuft the pendulum clock to apparent time.

18th March, 1783.	Sun's altit.	Ift obfer-vation.	Sun's altit.	IId obfer-vation.	Sun's altit.	IIId obfer-vation.	Sun's altit.	IVth ob-fervation.	Sun's altit.	Vth obfer-vation.
The fun's upper limb at the horizontal wire, eaftern fide	° ′ 25 40	h. ′ ″ 8 25 2	° ′ 26 10	h. ′ ″ 8 28 25	° ′ 26 40	h. ′ ″ 8 31 43	° ′ 27 20	h. ′ ″ 8 36 16	° 28	h. ′ ″ 8 40 53
Sun's upper limb at the fame altitude, weftern fide -		15 12 24		15 9 1		15 5 39		15 1 10		14 56 29
Dividing the fum by 2		23 37 26		23 37 26		23 37 22		23 37 26		23 37 22
Sun's center on the meridian as marked by the clock		11 48 43		11 48 43		11 48 41		11 48 43		11 48 41
Equation of the day		12 8 15		12 8 15		12 8 15		12 8 15		12 8 15
Clock flower than equated folar time		0 19 32		0 19 32		0 19 34		0 19 32		0 19 34

The mean of which I put 19′ 32″

19th

19th March, 1783.	Sun's Altit.	Ist observation.	Sun's Alt.	IId observation.
	° ′	h. ′ ″	°	h. ′ ″
The sun's upper limb at the horizontal wire of the first telescope on the eastern side	31 30	9 1 56	33	9 13 9
Sun's upper limb at the same altitude on the western side of the meridian		14 31 47		14 20 33
Dividing the sum by 2		23 33 43		23 33 42
Sun's center on the meridian as marked by the pendulum clock		11 46 51½		11 46 51
Equation of the day		12 7 57		12 7 57
Clock slower than equated solar time		0 21 6		0 21 6

		′	″
Clock slower than equated solar time	19th March	21	6
—— —— ——	18th March	19	32
Retarding of the clock upon 23 h. 58′ 8″ -		1	34

I observed too the mid-day at the great gnomon of the observatory, and found at the same time the meridian line erroneous by 19″, as you will find in the following tables.

		h. ′ ″		h. ′ ″		h. ′ ″
When the center of the sun's image was on the meridian the time pointed by the clock was	the 17th Mar.	11 50 50	the 18th Mar.	11 48 56	the 19th Mar.	11 47 3
Equations of those days -		12 8 33		12 8 15		12 7 57
Retarding on equated solar time		17 43		19 19	the 18th	20 54
Retarding the 17th - -				17 43		19 19
Retarding of the clock during those 24 hours				1 36		1 35

I fixed therefore the retarding of the clock 1′ 35″.

True

		h. ′ ″		h. ′ ″
True mid-day concluded by the sun's correspondent altitudes as the clock marked	the 18th Mar.	11 48 42	the 19th Mar.	11 46 51
Equation of the mid-day - -		— 18		— 18
Retarding of the clock at the rate of 1′ 35″ per 24 hours - -		+ 13		+ 11
True mid-day the pendulum clock should have marked - - - -		11 48 37		11 46 44
Mid-day concluded at the gnomon of the observatory - - - -		11 48 56		11 47 3
Difference, the error of the meridian line or gnomon - - -		19		19

From thence I concluded,

	h. ′ ″		h. ′ ″
Mid-day at true solar time	11 59 60	Mid-day at equated solar time	12 8 15
Mid-day the clock should have marked on the 18th	11 48 37	Mid-day the clock should have marked on the 18th	11 48 37
Retarding upon true solar time - -	11 23	Retarding upon equated solar time -	19 38

Observations

Observations of the moon's eclipse the 18th March, 1783.

My observations with an achromatic telescope of 3½ feet length.	Time marked by the clock.	True or apparent time.
IMMERSIONS.		
The beginning of the eclipse very doubtful.	h. ′ ″	h. ′ ″
Shadow touches Grimaldi	7 41 45	7 53 39
Grimaldi all in the shadow	7 42 54	7 54 48
Shadow touches Mare Humorum	7 49 32	8 1 27
——— Copernicus	8 0 21	8 12 16
Copernicus all in the shadow	8 2 29	8 14 24
Tycho touches the shadow	8 6 18	8 18 13
Eudoxus ———	8 12 19	8 24 15
——— all in the shadow	8 13 43	8 25 39
Mare Serenitatis touches the shadow	8 16 7	8 28 3
——— the shadow in the middle	8 21 31	8 33 27
——— all in the shadow	8 25 54	8 37 50
Proclus touches the shadow	8 32 21	8 44 18
Mare Crisium touches the shadow	8 33 29	8 45 26
——— shadow in the middle	8 35 36	8 47 33
——— all in the shadow	8 36 56	8 48 53
Total immersion	8 38 57	8 50 55
EMERSIONS.		
Beginning of the emersion	10 19 57	10 32 2
Grimaldi emerging	10 23 33	10 35 38
——— all out of the shadow	10 24 9	10 36 14
Mare Humorum emerging	10 29 34	10 41 39
Total emersion of Mare Humorum	10 35 37	10 47 43
Copernicus all out of the shadow	10 43 6	10 55 12
Mare Serenitatis all emerged	10 57 32	11 9 39
Mare Crisium all emerged	11 15 44	11 27 51
End of the eclipse	11 20 10	11 32 18
Total duration		3 39 0

Father

Father LE FEVRE's obſervations with a reflector 55 inches focal length, magnifying 300 times.

	Time by the clock.			Apparent time.		
IMMERSIONS.	h.	′	″	h.	′	″
Grimaldi touches the ſhadow - -	7	41	43	7	53	37
Kepler touches the ſhadow - -	7	52	2	8	3	57
——— all in the ſhadow - -	7	53	24	8	5	19
Copernicus touches the ſhadow - -	8	0	22	8	12	17
——— all in the ſhadow - -	8	2	26	8	14	21
Mare Serenitatis touches the ſhadow -	8	16	7	8	28	3
——— all in the ſhadow - -	8	26	2	8	37	58
Mare Criſium touches the ſhadow - -	8	33	28	8	45	25
——— all in the ſhadow - -	8	36	56	8	48	53
Total immerſion - - -	8	38	54	8	50	52
EMERSIONS.						
Beginning of the emerſion - -	10	19	42	10	31	47
Grimaldi emerged - -	10	23	24	10	35	29
Kepler all out of the ſhadow - -	10	35	43	10	47	49
Copernicus all out - - -	10	43	4	10	55	10
Mare Serenitatis all out - - -	10	57	19	11	9	26
—— Criſium all out - - -	11	15	50	11	27	57
End of the eclipſe - - -	11	20	22	11	32	30
Total duration - - -				3	39	20

The obſervation of the vernal equinox was made at the gnomon. The height of this gnomon, taken from the center of the hole by which the beams of the ſun come in, is 1878 lines of a French inch; the diſtance from the bottom of the gnomon to the equinoctial point is 1928; the diſtance from the upper limb of the ſun's image to the equinoctial point was found 16,7; the diſtance from the under limb 23,4; the diameter of the hole = 6; therefore the diſtance from the bottom to the upper limb 1928 − 16,7 + 3 = 1914,3, to the under limb 1928 + 23,4 − 3 = 1948,4; which gives the time the equinox happened the 20th of March, 5 h. 56′ 52″ P.M.

 Obſervations

Obſervations of Jupiter's ſatellites at Marſeilles.

1782		Apparent time.	Obſerva-tion.
		h. ′ ″	
April 3	Immerſion of the Iſt ſatellite at -	2 22 56	good
May 19	Imm. Iſt ſat. - - -	2 48 12	good
June 7	Imm. IVth ſat. was not total, but its light diminiſhed ſenſibly without ever diſappearing; the ſky was ſerene, and Jupiter had ſix belts very diſtinctly.		
20	Emerſion of the Iſt ſatellite - -	1 29 46	good
July 5	Em. Iſt ſat. - - -	11 43 59	good
13	Em. IId ſat. - - -	9 17 28	good
20	Em. IIId ſat. - - -	9 27 13	good
20	Em. IId ſat. - - -	11 51 59	good
21	Em. Iſt ſat. - - -	10 1 11	good
27	Imm. IIId ſat. - - -	10 40 33	good
Aug. 6	Em. Iſt ſat. - - - -	8 21 20	good
13	Em. Iſt ſat. - - -	10 18 49	good
14	Em. IId ſat. - - -	8 55 39	good
Sept. 1	Em. IIId ſat. - - -	9 40 44	doubtful
14	Em. Iſt ſat. - - -	8 6 48	good

IT is known, that the indirect method to calculate the orbits of comets in a conic ſection, by means of three obſervations given, is rendered more eaſy and expeditious if there is a poſſibility of drawing a graphical figure that repreſents nearly the orbit under conſideration, by means of which the calculation is directed, and the required elements of the comet's path may be rigorouſly determined. To draw the orbit of a comet that moves in a parabola or ellipſis, the problem is reduced to find the poſition of the axis and the perihelial diſtance; this poſition of the axis will be determined as ſoon as the angle is known, that the axis forms with another line, whoſe poſition is given; this line may be an ordinate to a given point of the curve, or a tangent, or a radius vector, &c. The latter is to

 be

be employed in preference, because the perihelial distance being a constant quantity, the angle of position then becomes the true anomaly of the comet; but as the data of this problem are only geocentric longitudes and latitudes of the comet, deduced from the immediate observations of right ascension and declination, the heliocentric longitudes and latitudes must first be calculated; but as those data are not sufficient, what is not given must be arbitrarily supposed, *viz.* the shortened distances (*distantias curtatas*). This supposition is changed and altered until the calculation will agree with the three observations, then the difference between two longitudes is the angle comprehended between the two shortened distances in the plane of the ecliptic; the whole reduced to the plane of the comet's orbit by means of the heliocentric latitude, gives the difference between the anomalies comprehended by two radius vectors, the problem then is reduced: two radius vectors being given, with the angle comprehended, to find the two true anomalies, the perihelial distance, and the time the comet puts in running its anomalies.

Let therefore ♈ ♋ ♎ ♑ represent the ecliptic at an infinite distance; QPR the apparent elliptical or parabolical path of a comet; S the sun's center; P the comet's perihelion; T the place of the earth when the comet was first observed in C; I the earth's place when the comet was observed in K; ST $= d$, SI $= \delta$, the distances from the earth to the sun at the first and second observation known by astronomical tables; let Cm and Kn be two perpendiculars to the plane of the ecliptic, it will be S$m = u$, S$n = v$ the two shortened distances.

The observed geocentric longitude of the comet in T $= a =$ arc ♈ ♑ G;
the observed geocentric longitude of the comet in I $= \alpha =$ arc ♈ ♑ H;
the geocentric longitude of the sun by tables in T $= b =$ arc ♈ ♑ ♎ A;
the geocentric longitude of the sun by tables in I $= \beta =$ arc ♈ ♑ ♎ B.

 Now

Now for the first observation the angle of elongation is $b - a$; for the angle ATG = arc AG = ♈ ♑ ♎ A − ♈ ♑ G = long. ☉ − long. comet. $= b - a$;

the angle of the annual parallax $SmT = \frac{\text{fin.}(b-a)d}{u} = e$;

the angle of commutation $mST = {}^{0^\circ}_{180^\circ} - e + (b - a) = f$;

from whence the heliocentric longitude of the comet $= b - 180^\circ + f = g$.

The same at the second observation in I.

Angle of elongation $= \beta - \alpha$;

Angle of annual parallax $\varepsilon = \frac{\text{fin.}(\beta-\alpha)\,\delta}{v}$;

Angle of commutation $\phi = {}^{0^\circ}_{180^\circ} - \varepsilon + (\beta - \alpha)$;

heliocentric longitude of the comet in $I = \beta - 180^\circ + \phi = \gamma$;

putting now the heliocentric latitude seen from $S = k$;

the geocentric latitude seen from $T = l$;

the heliocentric latitude will be $\frac{\text{fin.} f \,.\, \text{tang.} l}{\text{fin.} (b-a)} = \text{tang.} k$;

the same with Kn it will be $\frac{\text{fin.} \phi \,.\, \text{tang.} \lambda}{\text{fin.} (\beta-\alpha)} = \text{tang.} \varkappa$ heliocentric latitude in K.

Having thus determined the heliocentric latitudes of two observations, the radius vectors will easily be found in the supposition made for the shortened distances, for they are in the same ratio to the radius vectors as the cosine of the heliocentric latitudes are to the radius $= 1$; therefore the radius vector m of the first observation will be $= \frac{u}{\text{cof.} k}$ and the radius vector of the second observation $\mu = \frac{v}{\text{cof.} \varkappa}$.

Taking now the difference between the found heliocentric longitudes, we get the heliocentric motion of the comet upon the ecliptic between two shortened distances, which is to be reduced upon the comet's orbit, this heliocentric motion is therefore $\gamma - g = m$. Now to reduce this motion we have, first,

finus

ſinus totus $= 1$ is to coſine m :: as cotangent k is to the tangent of an angle which I put $= n$, and $90^\circ - k \mp n$ will give an angle which I put $= q$. Laſtly, the analogy coſ. n : coſ. q :: ſin. k : will give the coſine of an angle ψ, which is the required motion upon the orbit, or the angle comprehended between the two radius vectors m and μ. Let therefore ECPMND be the apparent parabolic path of a comet; S the ſun's center; M and N two places of the comet, the angle MSN equal to its motion in longitude, or the comprehended angle ψ; P the perihelion; it is required to find the two anomalies PM, PN, that is, PSM and PSN, the perihelial diſtance SP, and the time the comet employed to come from its perihelion P to M and N.

Reſolution.

$SM = m$	In the right-angled triangle SMR and SNV we have
$SN = \mu$	$MR = OS = m \text{ sin.} (\psi \pm x)$ $NV = QS = \mu \text{ sin.} x$;
$MSN = \psi$	therefore $OP = \frac{1}{4}p - m (\text{sin.} \psi \pm x)$ and $PQ =$
$NSB = x$	$\frac{1}{4}p \mp \mu \text{ sin.} x$; but by the nature of the parabola
$MSB = (\psi \pm x)$	we have $SM = AP + PO$ and $SN = AP + PQ$; that is
Parameter $= p$	$m = \frac{1}{2}p - m (\text{sin.} \psi \pm x)$ $\mu = \frac{1}{2}p \mp \mu \text{ sin.} x$

$$m + m (\text{sin.} \psi \pm x) = \tfrac{1}{2} p \qquad \mu \pm \mu \text{ sin.} x = \tfrac{1}{2} p$$

$$m (1 + \text{sin.} \psi \pm x) = \tfrac{1}{2} p \qquad \mu (1 \mp \text{sin.} x) = \tfrac{1}{2} p$$

and $1 + \text{sin.} (\psi \pm x) = \frac{p}{2m}$ $\qquad 1 \pm \text{sin.} x = \frac{p}{2\mu}$; by putting into a ſum $1 + \text{sin.} (\psi \pm x) + 1 \pm \text{sin.} x = \frac{p}{2m} + \frac{p}{2\mu}$; reduction made $2 \pm \text{sin.} x + \text{sin.} (\psi \pm x) = \left(\frac{m + \mu}{2m\mu}\right) p$; but by trigonometrical formulæ we have ſin. $(\psi \pm x) = \text{sin.} \psi \text{ cos.} x \pm \text{sin.} x$ coſ. ψ. Subſtituting this expreſſion in its place we obtain, $2 \pm \text{sin.} x + \text{sin.} \psi \text{ cos.} x \pm \text{sin.} x \text{ cos.} \psi = \left(\frac{m + \mu}{2m\mu}\right) p$. By the ſame formulæ we have $\text{cos.}^2 x = 1 - \text{sin.}^2 x$ and $\text{cos.} x = \sqrt{1 - \text{sin.}^2 x}$.

 Sub-

Subſtituting it comes out, $2 \pm \text{ſin.}\, x + \text{ſin.}\, \psi \sqrt{1 - \text{ſin.}^2 x} \pm \text{ſin.}\, x \,\text{coſ.}\, \psi = \left(\frac{m+\mu}{2m\mu}\right) p$ and $\text{ſin.}\, \psi \sqrt{1 - \text{ſin.}^2 x} = \left(\frac{m+\mu}{2m\mu}\right) p - 2 \mp \text{ſin.}\, x \mp \text{ſin.}\, x \,\text{coſ.}\, \psi$. By caſting away the ſign of the ſquare root, and reducing to o,

$$\left(\frac{m+\mu}{4m^2\mu^2}\right)^2 p^2 + 4 + \text{ſin.}^2 x + \text{ſin.}^2 x \,\text{coſ.}^2 \psi - 2\left(\frac{m+\mu}{m\mu}\right) p \mp \left(\frac{m+\mu}{m\mu}\right) p \,.\, \text{ſin.}\, x \mp \left(\frac{m+\mu}{m\mu}\right) p \,.\, (\text{ſin.}\, x \,\text{coſ.}\psi)$$

$$\pm 4 \,\text{ſin.}\, x \,\text{coſ.}\, \psi + 2 \,\text{ſin.}^2 x \,\text{coſ.}\, \psi - \text{ſin.}^2 \psi + \text{ſin.}^2 x \,\text{ſin.}^2 \psi = 0;$$

diſentangling the equation,

$$\frac{m^2p^2 + 2m\mu p^2 + \mu^2 p^2}{4m^2\mu^2} + 4 + \text{ſin.}^2 x + \text{ſin.}^2 x \,\text{coſ.}^2 \psi - \frac{2mp - 2\mu p}{m\mu} \mp \frac{mp \,\text{ſin.}\, x \mp \mu p \,\text{ſin.}\, x}{m\mu} \mp \frac{mp \,\text{ſin.}\, x \,\text{coſ.}\, \psi \mp \mu p \,\text{ſin.}\, x \,\text{coſ.}\, \psi}{m\mu}$$

$$\mp 4 \,\text{ſin.}\, x \pm 4 \,\text{ſin.}\, x \,\text{coſ.}\, \psi + 2 \,\text{ſin.}^2 x \,\text{coſ.}\, \psi - \text{ſin.}^2 \psi + \text{ſin.}^2 x \,\text{ſin.}^2 \psi = 0.$$

But p being equal to $2\mu \pm 2\mu \,\text{ſin.}\, x$, ſubſtituting this value, we have

$$\frac{4m^2\mu^2 \pm 8m^2\mu^2 \,\text{ſin.}\, x + 4m^2\mu^2 \,\text{ſin.}^2 x + 8m\mu^3 + 8m\mu^3 \,\text{ſin.}^2 x \pm 16m\mu^3 \,\text{ſin.}\, x + 4\mu^4 + 4\mu^4 \,\text{ſin.}^2 x \pm 8\mu^4 \,\text{ſin.}\, x}{4m^2\mu^2} + 4 + \text{ſin.}^2 x +$$

$$+ \text{ſin.}^2 x \,\text{coſ.}^2 \psi - \frac{4m\mu \mp 4m\mu \,\text{ſin.}\, x - 4\mu^2 \mp 4\mu^2 \,\text{ſin.}\, x \mp 2m\mu \,\text{ſin.}\, x - 2m\mu \,\text{ſin.}^2 x \mp 2\mu^2 \,\text{ſin.}\, x - 2\mu^2 \,\text{ſin.}^2 x \mp}{m\mu}$$

$$\frac{\mp 2m\mu \,\text{ſin.}\, x \,\text{coſ.}\psi - 2m\mu \,\text{ſin.}^2 x \,\text{coſ.}\psi \mp 2\mu^2 \,\text{ſin.}\, x \,\text{coſ.}\psi - 2\mu^2 \,\text{ſin.}^2 x \,\text{coſ.}\psi}{m\mu} \pm 4 \,\text{ſin.}\, x \pm 4 \,\text{ſin.}\, x \,\text{coſ.}\psi + 2 \,\text{ſin.}^2 x \,\text{coſ.}\psi - \text{ſin.}^2 \psi +$$

$$+ \text{ſin.}^2 x \,\text{ſin.}^2 \psi = 0.$$

Dividing the numerator and denominator of the firſt fraction by $4\mu^2$, and of the ſecond by μ, and putting the whole to the common denominator m^2 it comes

$$\frac{m^2 \pm 2m^2 \,\text{ſin.}\, x + m^2 \,\text{ſin.}^2 x + 2m\mu + 2m\mu \,\text{ſin.}^2 x \pm 4m\mu \,\text{ſin.}\, x + \mu^2 + \mu^2 \,\text{ſin.}^2 x \pm 2\mu^2 \,\text{ſin.}\, x + 4m^2 + m^2 \,\text{ſin.}^2 x \,\text{coſ.}^2 \psi + m^2 \,\text{ſin.}^2 x -}{m^2}$$

$- 4m^2$

$$\frac{-4m^2 \mp 4m^2 \text{fin.}\,x - 4m\mu \mp 4m\mu\,\text{fin.}\,x \mp 2m^2\,\text{fin.}\,x - 2m^2\,\text{fin.}^2 x \mp 2m\mu\,\text{fin.}\,x - 2m\mu\,\text{fin.}^2 x \mp 2m^2\,\text{fin.}\,x\,\text{cof.}\,\psi - 2m^2\,\text{fin.}^2 x\,\text{cof.}\,\psi \mp}{m^2}$$

$$\frac{\mp 2m\mu\,\text{fin.}\,x\,\text{cof.}\,\psi - 2m\mu\,\text{fin.}^2 x\,\text{cof.}\,\psi \pm 4m^2\,\text{fin.}\,x \pm 4m^2\,\text{fin.}\,x\,\text{cof.}\,\psi + 2m^2\,\text{fin.}^2 x\,\text{cof.}\,\psi - m^2\,\text{fin.}^2\,\psi + m^2\,\text{fin.}^2 x\,\text{fin.}^2\psi}{m^2} = 0.$$

Casting away m^2, and reducing, we have

$$m^2 + \mu^2 + \mu^2\,\text{fin.}^2 x \pm 2\mu^2\,\text{fin.}\,x + m^2\,\text{fin.}^2 x\,\text{cof.}^2\psi - 2m\mu \mp 2m\mu\,\text{fin.}\,x\,\text{cof.}\,\psi - 2m\mu\,\text{fin.}^2 x\,\text{cof.}\,\psi - m^2\,\text{fin.}^2\psi + m^2\,\text{fin.}^2 x\,\text{fin.}^2\,\psi \mp$$
$$\mp 2m\mu\,\text{fin.}\,x \pm 2m^2\,\text{fin.}\,x\,\text{cof.}\,\psi = 0.$$

Putting together $\text{fin.}^2 x$ and $\text{fin.}\,x$

$$(\mu^2 + m^2\text{cof.}^2\psi - 2m\mu\,\text{cof.}\,\psi + m^2\,\text{fin.}^2\psi)\,\text{fin.}^2 x \pm (2\mu^2 \mp 2m\mu\,\text{cof.}\,\psi \mp 2m\mu \pm 2m^2\,\text{cof.}\,\psi)\,\text{fin.}\,x + m^2 - 2m\mu + \mu^2 - m^2\,\text{fin.}^2\psi = 0.$$

Substituting $\text{cof.}^2\psi = 1 - \text{fin.}^2\psi$ we recover a quadratic equation,

$$(\mu^2 + m^2 - 2m\mu\,\text{cof.}\psi)\,\text{fin.}^2 x \pm (2\mu^2 \mp 2m\mu\,\text{cof.}\psi \pm 2m^2\text{cof.}\psi \mp 2m\mu)\,\text{fin.}\,x + m^2 - 2m\mu + \mu^2 - m^2\,\text{fin.}^2\psi = 0,$$

and $\text{fin.}^2 x \pm \dfrac{(2\mu^2 \mp 2m\mu\,\text{cof.}\,\psi \pm 2m^2\text{cof.}\,\psi \mp 2m\mu)}{(\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi)}\,\text{fin.}\,x = \dfrac{2m\mu - m^2 - \mu^2 + m^2\,\text{fin}^2\psi}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi}.$

This equation resolved gives

$$\text{fin.}\,x \pm \frac{\mu^2 \mp m\mu\,\text{cof.}\,\psi \pm m^2\,\text{cof.}\,\psi \mp m\mu}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi} = \sqrt{\frac{2m\mu - m^2 - \mu^2 + m^2\,\text{fin.}^2\psi}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi} \pm \frac{(\mu^2 \mp m\mu\,\text{cof.}\,\psi \pm m^2\,\text{cof.}\,\psi \mp m\mu)^2}{(\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi)^2}}$$ which gives farther,

$$\text{fin.}\,x = \frac{\sqrt{(2m\mu - m^2 - \mu^2 + m^2\text{fin.}^2\psi)\,.\,(\mu^2 + m^2 - 2m\mu\text{cof.}\psi) \pm (\mu^2 \mp m\mu\text{cof.}\psi \pm m^2\text{cof.}\psi \mp m\mu)^2}}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi} \mp \frac{\mu^2 \mp m\mu\text{cof.}\psi \pm m^2\text{cof.}\psi \mp m\mu}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi}.$$

Putting $1 - \text{fin.}^2\psi$ instead of $\text{cof.}^2\psi$, after all reduction made, we recover

$$\text{fin.}\,x = \frac{\sqrt{2m^3\mu\,\text{fin.}^2\psi - 2m^3\mu\,\text{cof.}\,\psi\,\text{fin.}^2\psi}}{m^2 + \mu^2 - 2m\mu\,\text{cof.}\,\psi} \pm \left(\mp \frac{\mu^2 \pm \mu m\,\text{cof.}\,\psi \mp m^2\,\text{cof.}\,\psi \pm m\mu}{\mu^2 + m^2 - 2m\mu\,\text{cof.}\,\psi}\right);$$ that is,

$\text{fin.}\,x = (\pm\mu \mp m)\,.\,(m\,\text{cof.}\,\psi - \mu) \pm \sqrt{2m^3\mu\,\text{fin.}^2\psi\,(1 - \text{cof.}\,\psi)}$, which gives at last the simple expression $\text{fin.}\,x = \dfrac{(\pm\mu \mp m)\,.\,(m\,\text{cof.}\,\psi - \mu) \pm m\,\text{fin.}\,\psi\sqrt{2m\mu\,(1 - \text{cof.}\,\psi)}}{m^2 + \mu^2 - 2m\mu\,\text{cof.}\,\psi}.$

The angle x defines therefore the position of the axis and the two anomalies required, the perihelial distance being $p = 2\mu \pm 2\mu$ sin. x, it will be known also by the angle x.

In order to find the time the comet employs in running its anomalies, let the perihelial distance just now investigated p be equal to the radius of the earth's orbit, the parabolic area swept by the radius vector will be by the nature of the parabola $\frac{2}{3}PO \times OM + \frac{1}{2} SO \times OM = \frac{4PO \times OM + 3SO \times OM}{6}$. Now the periphery of the earth's orbit is $7 : 22 :: 2p : \frac{44}{7}p$; therefore the whole area $\frac{44}{7}p \cdot \frac{1}{2}p = \frac{22}{7}p^2$. It is known that the velocity of a heavenly body moved in a circular path, is to the velocity in a parabolic path in the ratio $\sqrt{2} : 1$. If the parabolic area of the comet is divided by $\sqrt{2}$ it comes out $\frac{4PO \times OM + 3SO \times MO}{6\sqrt{2}}$ equal to an area that the earth describes in the very same time; put therefore A equal to the time of a sidereal year, we shall recover the analogy; the whole area of the earth's orbit is to the time in which it is described as the parabolic area is to the time consumed in sweeping it; therefore $\frac{22}{7}p^2 : A :: \frac{(4PO + 3SO)\ MO}{6\sqrt{2}} : \frac{7A\ (4PO + 3SO)\ MO}{72p^2\sqrt{2}}$; but OM = SM . sin. anom. PSM and OS = SM . cos. anom. PSM; let the anomaly be $=\delta$, we have OM $= m$ sin. δ, and OS $= m$ cos. δ; therefore PO $= p - m$ cos. δ. Substituting we obtain $\frac{7A\ (4p - 4m \text{ cos. } \delta + 3m \text{ cos. } \delta)\ m \text{ sin. } \delta}{72p^2\sqrt{2}}$ which is $\frac{7A\ (4p - m \text{ cos. } \delta)\ m \text{ sin. } \delta}{72p^2\sqrt{2}}$, whereby the time is found in parts of a sidereal year.

I am, &c.

SIR,

SIR, Lyons, May 4, 1783.

LATELY I received from the Obſervatory at Marſeilles the obſervation of the tranſit of Mercury, which happened the 12th Nov. 1782. The ſky not being very favourable, only the two internal contacts were obſerved; the firſt internal contact was obſerved by M. ST. JACQUES DE SYLVABELLE, at 3 h. 18′ 30″ apparent time; the laſt internal contact by M. ST. JACQUES, at 4 h. 30′ 16″; by M. BERNARD, his Adjunctus, at 4 h. 29′ 13″. The neareſt diſtances of Mercury's limb to that of the ſun in the northern part of its diſk were at

h.	′	″		
3	33	14	31	parts of the micrometer.
3	42	57	34	
4	22	17	19	

The apparent diameter of the ſun was 2174 parts of this micrometer: I ſuppoſe the before-mentioned 2174 parts = 32′ 26″,9. I conclude farther, by the obſervations, the middle of the tranſit at 3 h. 54′ 7″,25, whereas I fix, by interpolation, the diſtances of the limbs at 3 h. 54′ 7″,25 = 35″,6; I have therefore ſemi-diameter of the ſun = 16′ 13″,4 − 35″,6 = 15′ 37″,8 + ſemi-diameter of Mercury = 6″ = 15′ 43″,8 = to the leaſt diſtance of centers of the ſun and Mercury. By M. DE LA LANDE's tables it is 15′ 42″, only a difference of 1″,8.

M. WALLOT at Paris has obſerved this tranſit at the Royal Obſervatory,

	h.	′	″
Firſt external contact	2	56	28
Firſt internal contact	3	2	3
Second - -	4	17	18
Second external -	4	22	53

I only add an important remark upon the diameter of Mercury, which the astronomers supposed in this transit = 12″.

Let ABC represent the sun's disk; in P an external in Q an internal contact; ANC the apparent path of Mercury over the sun.

The semi-diameter of the sun = 972″, this of Mercury in our supposition = 6″, MN = 942″ the least distances of the centers. In the right-angled triangle MNP it is MP = 972″ + 6″ = 978″, MQ = 972″ − 6″ = 966″; therefore NP will be found = 260″ and NQ = 210″: now NP − NQ = PQ = 50″, which converted into time gives 8′ 14″ for the time the diameter of Mercury employed to run over the limb of the sun; but by the observations of M. WALLOT I find this time constantly in both contacts 5′ 35″; therefore 8′ 14″ : 12″ :: 5′ 35″ : 8″,137, which should be the diameter of Mercury; and indeed M. WALLOT, by an immediate measure, taken with an excellent wire-micrometer, finds this apparent diameter not greater than 9″, which sufficiently shews that this diameter supposed 7″ in the mean distance is also too great.

I am, &c.

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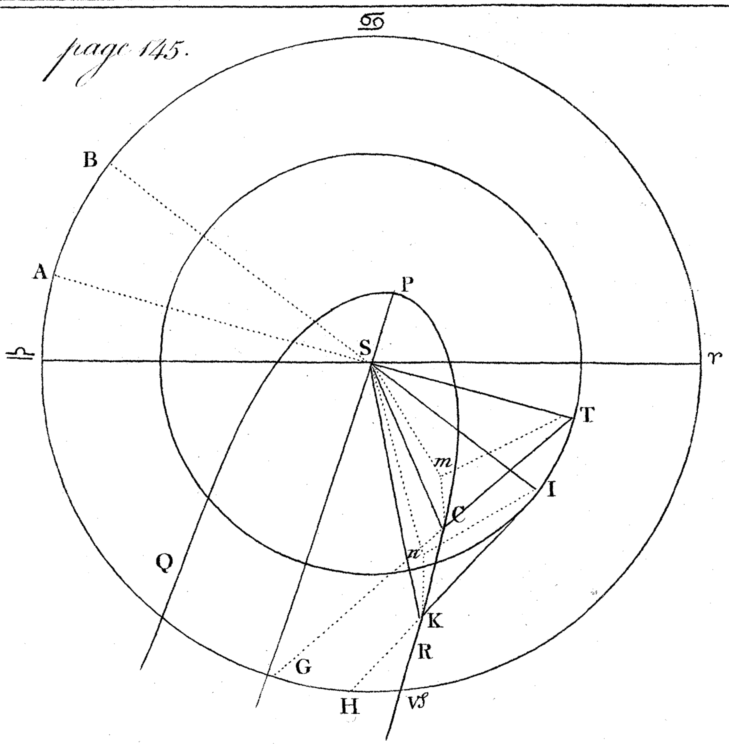

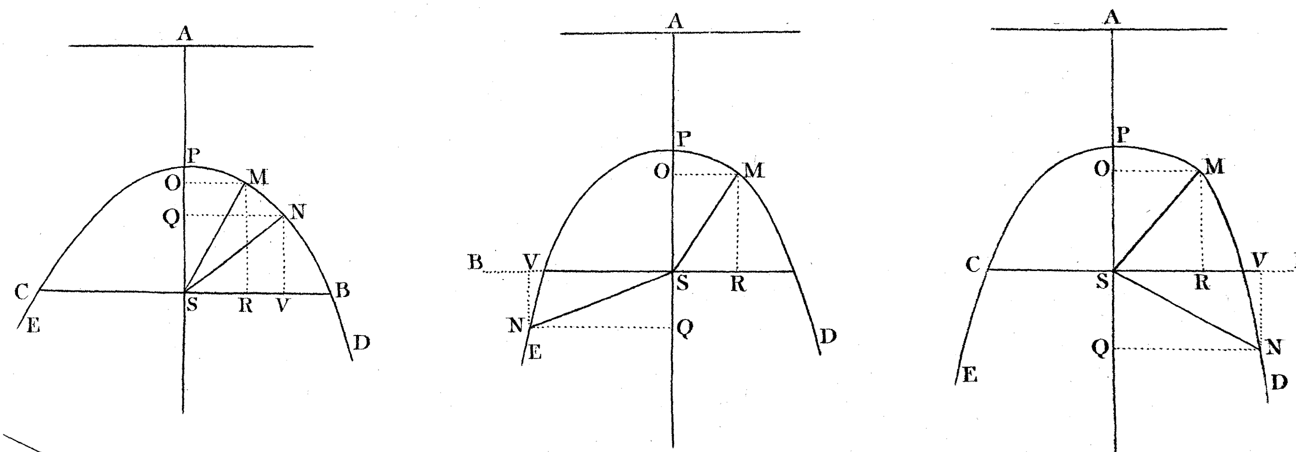

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